

# Pulsar Constraints on Neutron Star Structure and Equation of State

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With the aim of constraining the structural properties of neutron stars and the equation of state of dense matter, we study sudden spin-ups, *glitches*, occurring in the Vela pulsar and in six other pulsars. We present evidence that glitches represent a self-regulating instability for which the star prepares over a waiting time. The angular momentum requirements of glitches in Vela indicate that  $\geq 1.4\%$  of the star's moment of inertia drives these events. If glitches originate in the liquid of the inner crust, Vela's 'radiation radius'  $R_\infty$  must exceed  $\simeq 12$  km for a mass of  $1.4M_\odot$ . Observational tests of whether other neutron stars obey this constraint will be possible in the near future.

## I. INTRODUCTION

The sudden spin jumps, or *glitches*, commonly seen in isolated neutron stars are thought to represent angular momentum transfer between the crust and the liquid interior [1]. In this picture, as a neutron star's crust spins down under magnetic torque, differential rotation develops between the stellar crust and a portion of the liquid interior. The more rapidly rotating component then acts as an angular momentum reservoir which occasionally exerts a spin-up torque on the crust as a consequence of an instability. The Vela pulsar, one of the most active glitching pulsars, typically undergoes fractional changes in rotation rate of  $\sim 10^{-6}$  every three years on average [2]. With the Vela pulsar having exhibited 13 glitches, meaningful study of the statistical properties of these events is now possible.

In this Letter we study the time distribution of Vela's glitches and determine the average angular momentum transfer rate in Vela and in six other pulsars. We present evidence that glitches in Vela represent a self-regulating instability for which the star prepares over a waiting interval. We obtain a lower limit on the fraction of the star's liquid interior responsible for glitches. Assuming that glitches are driven by the liquid residing in the inner crust, as in most glitch models, we show that Vela's 'radiation radius' is  $R_\infty \gtrsim 12$  km for a mass of  $1.4M_\odot$ . Future

measurements of neutron star radii will check the universality of this constraint and hence test our understanding of neutron star structure and the origin of glitches.

## II. REGULARITY OF ANGULAR MOMENTUM TRANSFER

A glitch of magnitude  $\Delta\Omega_i$  requires angular momentum

$$\Delta J_i = I_c \Delta\Omega_i, \quad (1)$$

where  $I_c$  is the moment of inertia of the solid crust plus any portions of the star tightly coupled to it. Most of the core liquid is expected to couple tightly to the star's solid component, so that  $I_c$  makes up at least 90% of the star's total moment of inertia [3]. Glitches are driven by the portion of the liquid interior that is differentially rotating with respect to the crust. The cumulative angular momentum imparted to the crust over time is

$$J(t) = I_c \bar{\Omega} \sum_i \frac{\Delta\Omega_i}{\bar{\Omega}}, \quad (2)$$

where  $\bar{\Omega} = 70.4 \text{ rad s}^{-1}$  is the average spin rate of the crust over the period of observations. Fig. 1 shows the cumulative dimensionless angular momentum,  $J(t)/I_c\bar{\Omega}$ , over  $\sim 30$  years of glitch observations of the Vela pulsar, with a linear least-squares fit. The average rate of angular momentum transfer associated with glitches is  $I_c\bar{\Omega}A$ , where  $A$  is the slope of the straight line in Fig. 1:

$$A = (6.44 \pm 0.19) \times 10^{-7} \text{ yr}^{-1}. \quad (3)$$

This rate  $A$  is often referred to as the *pulsar activity parameter*.

The angular momentum flow is extremely regular; none of Vela's 13 glitches caused the cumulative angular momentum curve to deviate from the linear fit shown in Fig. 1 by more than 12%. To assess the likelihood that the linear trend could have arisen by chance, we tested the statistical robustness of this result. We generated many sets of simulated data in which the occurrence

times of the glitches remained as observed, but the magnitudes of the 13 glitches were randomly shuffled. We compared the observed  $\chi^2$  to those for the deviations of the randomly shuffled data from linear fits. The  $\chi^2$  for the shuffled data was less than that of the real  $\chi^2$  in only  $\sim 1.4\%$  of cases, strongly suggesting that the rate of angular momentum flow associated with glitches is reasonably constant.

Additionally, the near uniformity of the intervals between the glitches in Fig. 1 suggests that glitches occur at fairly regular time intervals. The standard deviation in observed glitch intervals is  $0.53\langle\Delta t\rangle$ , where  $\langle\Delta t\rangle = 840$  d is the average glitch time interval. The probability of 13 randomly-spaced (Poisson) events having less than the observed standard deviation is only  $\sim 1\%$ .

The data of Fig. 1 indicate that Vela's glitches are not random, but represent a self-regulating process which gives a relatively constant flow of angular momentum to the crust with glitches occurring at fairly regular time intervals.

### III. THE GLITCH RESERVOIR'S MOMENT OF INERTIA

The average rate of angular momentum transfer in Vela's glitches constrains the properties of the angular momentum reservoir that drives the spin jumps. In particular, the frequent occurrence of large glitches requires that a significant fraction of the interior superfluid spins at a higher rate than the crust of the star. Between glitches, the reservoir acquires excess angular momentum as the rest of the star slows under the magnetic braking torque acting on the crust. Excess angular momentum accumulates at the maximum rate if the reservoir does not spin down between glitches. Hence, the rate at which the reservoir accumulates angular momentum capable of driving glitches is limited by

$$\dot{J}_{\text{res}} \leq I_{\text{res}}|\dot{\Omega}|, \quad (4)$$

where  $\dot{\Omega}$  is the average spin-down rate of the crust, and  $I_{\text{res}}$  is the moment of inertia of the angular momentum reservoir (not necessarily one region of the star). Equating  $\dot{J}_{\text{res}}$  to the average rate of angular momentum transfer to the crust,  $I_c\dot{\Omega}A$ , gives the constraint,

$$\frac{I_{\text{res}}}{I_c} \geq \frac{\bar{\Omega}}{|\dot{\Omega}|} A \equiv G, \quad (5)$$

where the *coupling parameter*  $G$  is the minimum fraction of the star's moment of inertia that stores angular momentum and imparts it to the crust in glitches. Using the observed value of Vela's activity parameter  $A$  and  $\bar{\Omega}/|\dot{\Omega}| = 22.6$  Kyr, we obtain the constraint

$$\frac{I_{\text{res}}}{I_c} \geq G_{\text{Vela}} = 1.4\%. \quad (6)$$

A similar analysis for six other pulsars yields the results shown in Fig. 2. An earlier analysis of glitches in Vela gave  $I_{\text{res}}/I_c \geq 0.8\%$  [4]. After Vela, the most significant limit is obtained from PSR 1737-30 which gives  $I_{\text{res}}/I_c \geq G_{1737} = 1\%$ .

The similarity of  $G$  for the five objects of intermediate age suggests that glitches in all these objects are driven by internal components with about the same fractional moment of inertia. In terms of  $G$ , the Crab pulsar and PSR 0525+21 appear to be unusual. It may be that the Crab's angular momentum reservoir loses its excess angular momentum between glitches, perhaps through thermal creep of superfluid vortices (see, e.g., [6]). The value of  $G$  for PSR 0525+21 is not well determined, since only two glitches from this object have been measured.

### IV. IMPLICATIONS FOR THE DENSE MATTER EQUATION OF STATE

The constraint of  $I_{\text{res}}/I_c \geq 1.4\%$  for Vela applies regardless of where in the star glitches originate. Many glitch models, however, assume that the internal angular momentum reservoir is the superfluid that coexists with the inner crust lattice [1], where the pinning of superfluid vortex lines sustains a velocity difference between the superfluid and the crust. Here we explore the implications of this interpretation. We begin by describing how the moment of inertia of the superfluid in the neutron star crust relates to the nuclear matter equation of state (EOS) and the observable properties of neutron stars.

Ravenhall & Pethick [7] have shown that, for various equations of state, the total moment of inertia  $I$  is given by the approximate expression

$$\left[1 + \frac{2GI}{R^3c^2}\right] I \simeq \frac{8\pi}{3} \int_0^R r^4(\rho + P/c^2)e^\lambda dr \equiv \tilde{J}, \quad (7)$$

where  $\rho$  is the mass-energy density,  $P$  is the pressure, and  $e^\lambda$  is the local gravitational redshift. This expression, which holds in the limit of slow rotation, defines the integral  $\tilde{J}$ . This integral can be evaluated following Lattimer & Prakash [8] who noted that  $\rho \propto 1 - (r/R)^2$  throughout most of the interior of a neutron star (but *not* in the crust), for all commonly-used equations of state. With this approximation, it can be shown [8] that

$$\tilde{J} \simeq \frac{2}{7}MR^2\Lambda, \quad (8)$$

where  $\Lambda \equiv (1 - 2GM/Rc^2)^{-1}$  and  $M$  is the total stellar mass.

Equation (7) can also be used to determine the moment of inertia of the crust plus liquid component. In the crust  $P$  is  $\ll \rho c^2$ , and the TOV equation is

$$\frac{dP}{dr} \simeq -GM\rho(r)\frac{e^\lambda}{r^2}. \quad (9)$$

Using this approximation in eq. (7) gives the fraction of the star's moment of inertia contained in the solid crust (and the neutron liquid that coexists with it):

$$\frac{\Delta I}{I} \simeq \frac{8\pi}{3\tilde{J}} \int_{R-\Delta R}^R \rho r^4 e^\lambda dr \simeq \frac{8\pi}{3\tilde{J}GM} \int_0^{P_t} r^6 dP. \quad (10)$$

Here  $\Delta R$  is the radial extent of the crust and  $P_t$  is the pressure at the crust-core interface. A similar approximation is obtained in Ref. 7 (equation 17); either approximation is adequate for the estimates we are making here. In most of the crust, the equation of state has the approximately polytropic form  $P \propto \rho^{4/3}$ , giving [8]

$$\int_0^{P_t} r^6 dP \simeq P_t R^6 \left[ 1 + \frac{8P_t}{n_t m_n c^2} \frac{4.5 + (\Lambda - 1)^{-1}}{\Lambda - 1} \right]^{-1}, \quad (11)$$

where  $n_t$  is the density at the core-crust transition and  $m_n$  is the neutron mass.  $\Delta I/I$  can thus be expressed as a function of  $M$  and  $R$  with an additional dependence upon the EOS arising through the values of  $P_t$  and  $n_t$ . However,  $P_t$  is the main EOS parameter as  $n_t$  enters chiefly via a correction term. In general, the EOS parameter  $P_t$  varies over the range  $0.25 < P_t < 0.65$  MeV fm $^{-3}$  for realistic equations of state [8]. Larger values of  $P_t$  give larger values for  $\Delta I/I$ , as can be seen from eq. [11].

Combining of eqs. [10] and [11] with a lower limit on  $\Delta I$  and an upper limit on  $P_t$  gives a lower limit on the neutron star radius for a given mass. In order to relate our observational bound on  $I_{\text{res}}/I_c$  to  $\Delta I$ , we assume that the angular momentum reservoir is confined to the neutron superfluid that coexists with the nuclei of the inner crust. In this case,  $I_{\text{res}} \leq \Delta I$  and  $I_c \geq I - \Delta I$ . Our observational limit on  $I_{\text{res}}$  then gives  $\Delta I/(I - \Delta I) \geq \Delta I/I_c \geq I_{\text{res}}/I_c \geq 0.014$ . To obtain a strong lower limit on the neutron star radius, we take  $P_t = 0.65$  MeV fm $^{-3}$  and  $n_t = 0.075$  fm $^{-3}$ . Combining the relations [10] and [11], gives the heavy dashed curve in Fig. 3. This curve is given approximately by

$$R = 3.6 + 3.9M/M_\odot. \quad (12)$$

Stellar models that are compatible with the lower bound on  $I_{\text{res}}$  must fall below this line. Smaller  $P_t$  reduces the crustal moment of inertia and gives a more restrictive constraint. For example,  $P_t = 0.25$  MeV fm $^{-3}$  moves the constraining contour to approximately  $R = 4.7 + 4.1M/M_\odot$  (thin dashed curve of Fig. 3).

## V. DISCUSSION

To summarize our conclusions regarding the statistics of Vela's glitches, we find that angular momentum is imparted to the crust at regular time intervals at a rate that has remained nearly constant for  $\sim 30$  yr. These data narrowly constrain the *coupling parameter*  $G$  which is the

minimum fraction of the star's moment of inertia that is responsible for glitches. For Vela we find  $G = 0.014$ , indicating that least 1.4% of the star's moment of inertia acts as an angular momentum reservoir for driving the glitches, regardless of where in the star this angular momentum reservoir is, or how it is coupled to the crust. Variation of  $G$  by a factor of less than  $\sim 3$  for stars in the age group  $10^4 - 10^5$  yr suggests that glitches in stars in this age group all involve regions of about the same fractional moment of inertia.

Mass measurements of radio pulsars in binary systems and of neutron star companions of radio pulsars give neutron star masses consistent with a very narrow distribution,  $M = 1.35 \pm 0.04 M_\odot$  [19], indicated by the pair of horizontal dotted lines in Fig. 3. If Vela's mass falls in this range, eq. [12] constrains  $R \gtrsim 8.9$  km, under the assumption that glitches arise in the inner crust superfluid. The quantity constrained by observations of the stellar luminosity and spectrum is the 'radiation radius'  $R_\infty \equiv \Lambda^{1/2} R = (1 - 2GM/Rc^2)^{-1/2} R$ . If  $M = 1.35 M_\odot$  for Vela, the above constraint gives  $R_\infty \gtrsim 12$  km if glitches arise in the inner crust. For comparison, we show in Fig. 3 the mass-radius curves for several representative equations of state (heavy solid lines). Measurement of  $R_\infty \lesssim 13$  km would be inconsistent with most equations of state if  $M \simeq 1.35 M_\odot$ . Stronger constraints could be obtained if improved calculations of nuclear matter properties indicate  $P_t$  significantly less than  $0.65$  MeV fm $^{-3}$ . For example, for  $M \simeq 1.35 M_\odot$ ,  $R_\infty \gtrsim 13$  km would be required if  $P_t = 0.25$  MeV fm $^{-3}$ . A measurement of  $R_\infty \lesssim 11$  km would rule out most equations of state regardless of mass or the angular momentum requirements of glitches.

A promising candidate for a decisive measurement of a neutron star's radiation radius is RX J185635-3754, an isolated, non-pulsing neutron star [13]. A black body fit to the X-ray spectrum gives  $R_\infty = 7.3(D/120 \text{ pc})$  km where  $D$  is the distance (known to be less than 120 pc). However, either a non-uniform surface temperature or radiative transfer effects in the stellar atmosphere could raise this estimate significantly [20]. HST observations planned for this year should determine the star's proper motion and parallax, and hence, the distance. Future CHANDRA observations should yield more detailed spectral data and could establish the composition of the atmosphere if absorption lines are identified. These distance and spectral data may establish whether this object's radius is consistent with an inner crust explanation of neutron star glitches.

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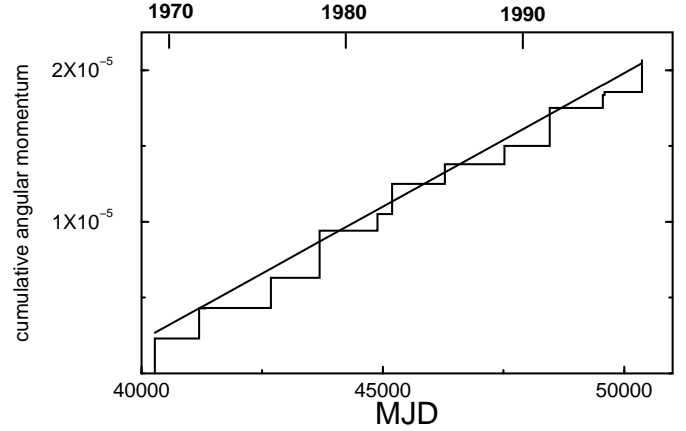


Fig. 1 – Cumulative dimensionless angular momentum,  $J/I_c\Omega$ , imparted to the Vela pulsar's crust as a function of time. The straight line is a least-squares fit.

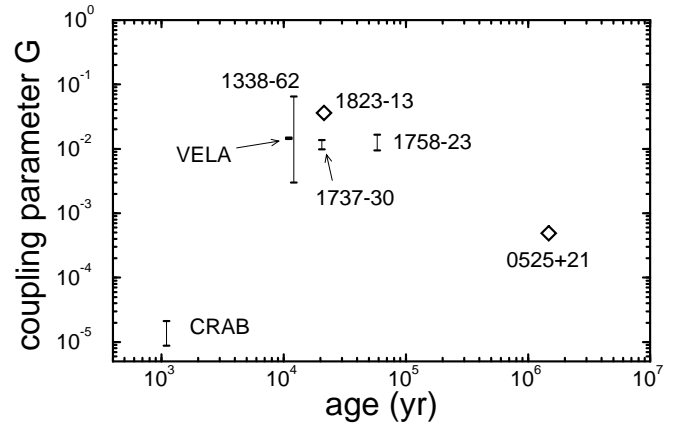


Fig. 2 – The coupling parameter  $G$ . The strongest constraints are obtained for Vela and PSR 1737-30, for which 13 and 9 glitches have been observed, respectively. Diamonds indicate objects with only two observed glitches, for which error bars could not be obtained. References: 0525+21 [14], Crab [15], Vela [2], 1338-62 [16], 1737-30 [17,18], 1823-13 [18].

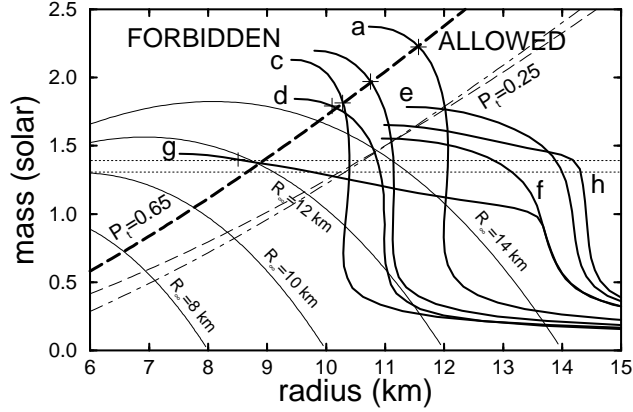


Fig. 3 – Limits on the Vela pulsar’s radius. The heavy dashed curve delimits allowed masses and radii that are compatible with the glitch constraint  $\Delta I / (I - \Delta I) \geq 1.4\%$  for  $P_t = 0.65 \text{ MeV fm}^{-3}$ . The thin dashed curve corresponds to  $P_t = 0.25 \text{ MeV fm}^{-3}$  and gives a more stringent though less conservative constraint. The dot-dashed curve corresponds to  $\Delta I / (I - \Delta I) \geq 2.8\%$  and  $P_t = 0.65 \text{ MeV}$ . The horizontal dashed lines indicate the mass limits for the survey of 26 radio pulsars of Ref. 19. Also displayed are mass-radius relations for the equations of state of Akmal & Pandharipande [10] (curves a and b), Wiringa, Fiks & Fabrocini [11] (curves c and d), Müller & Serot [12] (curves e and f) and the kaon EOS of Glendenning & Schaffner-Bielich [9] (curves g and h). The crosses indicate where a given EOS has  $\Delta I / (I - \Delta I) = 1.4\%$  (with  $P_t = 0.65 \text{ MeV fm}^{-3}$ ). Curves without crosses have  $\Delta I / (I - \Delta I) > 1.4\%$  for all stable  $R$ . Thin curves are contours of constant radiation radius  $R_\infty$ .